Benchmarking a system of time series: Denton's movement preservation principle vs. a data based procedure

Aggiustamento di un sistema di serie storiche: Denton vs. data based benchmarking

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Riassunto: In questo lavoro vengono poste a confronto due procedure di *benchmarking* di un sistema di serie storiche economiche, entrambe in grado di aggiustare le serie in input in modo da produrre serie che simultaneamente soddisfano vincoli di aggregazione temporale e contabile. La prima procedura estende in ambito multivariato l'approccio di Denton (1971), mentre la seconda (Guerrero e Nieto, 1999) tiene conto di eventuali covariazioni delle serie preliminari tramite un approccio basato sui modelli VAR. Vengono quindi presentati i risultati di due applicazioni empiriche, la prima condotta su dati simulati, la seconda su serie reali.

Keywords: benchmarking, movement preservation principle, VAR.

1. Introduction

Most of the data obtained by statistical agencies have to be adjusted, corrected or somehow processed by statisticians in order to arrive at useful, consistent and publishable values. As an example, the government agencies that collect and publish Quarterly National Accounts time series must produce subannual data that simultaneously comply with the relevant annual figures and satisfy accounting constraints (Eurostat, 1999). This kind of problem arises also when a system of time series is seasonally adjusted using a direct univariate approach, so that the accounting constraints valid for the raw series are not fulfilled (Di Fonzo and Marini, 2003).

Starting from a situation in which temporal and contemporaneously aggregated series are known, temporal (e.g., between monthly and annual data) and contemporaneous (between the monthly aggregate and the sum of its component series) discrepancies can be smoothed using benchmarking procedures. In this paper we consider (i) an extension of the univariate approach by Denton (1971), founded on a well known movement preservation principle, and (ii) a data-based benchmarking procedure (Guerrero and Nieto, 1999) which exploits the autoregressive properties of the preliminary series to be adjusted. In order to evaluate their performance in practical situations, both procedures are applied to simulated and real world data.

In the next section we state the problem and introduce some notation. The benchmarking procedures are briefly described in section 3, while the fourth section presents some summary statistics resulted from the applications.

2. Statement of the problem

Given M temporally aggregated (say, annual) time series ($\mathbf{y}_{0j}$, $j = 1, \ldots, M$) and a contemporaneously aggregated high–frequency (say, quarterly) time series ($\mathbf{z}$), suppose that M preliminary quarterly time series $\mathbf{p}_j$, $j = 1, \ldots, M$, are available. Vectors $\mathbf{y}_{0j}$, $\mathbf{z}$ and $\mathbf{p}_j$ have dimensions $(N \times 1)$, $(n \times 1)$ and $(n \times 1)$, respectively. Furthermore, it is $\mathbf{C}\mathbf{p}_j \neq \mathbf{y}_{0j}$, $j = 1, \ldots, M$, $\mathbf{C}$ being a $(N \times n)$ temporal aggregation matrix, and/or $\sum_{j=1}^{M} \mathbf{p}_j \neq \mathbf{z}$. Denoting by $\mathbf{y}_j$, $j = 1, \ldots, M$, the series to be estimated, which are $(n \times 1)$ vectors, and by $\mathbf{y}_0 = (\mathbf{y}'_{01}, \ldots, \mathbf{y}'_{0j}, \ldots, \mathbf{y}'_{0M})'$ the $(MN \times 1)$ vector of temporally aggregated series, the complete set of accounting constraints (both temporal and contemporaneous) can be written as $\mathbf{H}\mathbf{y} = \mathbf{y}_a$, where

$$\mathbf{H} = \begin{bmatrix} \mathbf{i}'_M \otimes \mathbf{I}_n \\ \mathbf{I}_m \otimes \mathbf{C} \end{bmatrix}, \qquad \mathbf{y} = (\mathbf{y}'_1, \ldots, \mathbf{y}'_j, \ldots, \mathbf{y}'_M)', \qquad \mathbf{y}_a = \begin{bmatrix} \mathbf{z} \\ \mathbf{y}_0 \end{bmatrix},$$

$\mathbf{i}_M$ is a $(M \times 1)$ vector of ones, and $\mathbf{H}$, $\mathbf{y}$ and $\mathbf{y}_a$ are $((n + MN) \times n(M+1))$, $(Mn \times 1)$ and $((n + MN) \times 1)$, respectively.

It can be shown that N linear restrictions of the $n + MN$ established by $\mathbf{H}\mathbf{y} = \mathbf{y}_a$ are redundant, and then matrix $\mathbf{H}$ has not full rank[1]. This fact has to be considered in developing benchmarked estimates $\hat{\mathbf{y}}_j$, $j = 1, \ldots, M$, such that $\mathbf{H}\hat{\mathbf{y}} = \mathbf{y}_a$, where, with obvious notation, it is $\hat{\mathbf{y}} = (\hat{\mathbf{y}}'_1, \ldots, \hat{\mathbf{y}}'_j, \ldots, \hat{\mathbf{y}}'_M)'$.

3. Two benchmarking procedures

The classical Denton's approach is grounded on a 'movement preservation principle', according to which the benchmarked estimates should have dynamics as near as possible to those of the preliminary ones. More precisely, the benchmarked estimates are obtained through minimization of the quadratic loss function

$$\sum_{j=1}^{M} \sum_{t=1}^{n} \left(\frac{\hat{y}_{j,t} - p_{j,t}}{p_{j,t}} - \frac{\hat{y}_{j,t-1} - p_{j,t-1}}{p_{j,t-1}} \right)^2 \equiv \sum_{j=1}^{M} \sum_{t=1}^{n} \left(\frac{\hat{y}_{j,t}}{p_{j,t}} - \frac{\hat{y}_{j,t-1}}{p_{j,t-1}} \right)^2$$

subject to $\mathbf{H}\mathbf{y} = \mathbf{y}_a$. The solution, expressed in terms of $r = n + (M-1)N$ 'free' observations[2], is the following:

$$\hat{\mathbf{y}} = \mathbf{p} + \boldsymbol{\Omega}\mathbf{H}'_w \boldsymbol{\Omega}_w^{-1} (\mathbf{y}_w - \mathbf{H}_w \mathbf{p}), \tag{1}$$

where $\mathbf{p} = (\mathbf{p}'_1, \ldots, \mathbf{p}'_j, \ldots, \mathbf{p}'_M)'$, $\mathbf{P} = diag\{\mathbf{p}_1, \ldots, \mathbf{p}_M\}$, and

$$\boldsymbol{\Omega} = \mathbf{P}\left[\mathbf{I}_M \otimes (\mathbf{D}'\mathbf{D})^{-1}\right]\mathbf{P}, \quad \mathbf{H}_w = \begin{bmatrix} \mathbf{i}'_{M-1} \otimes \mathbf{I}_n & \vdots & \mathbf{I}_n \\ \mathbf{I}_{M-1} \otimes \mathbf{C} & \vdots & \mathbf{0} \end{bmatrix}, \quad \boldsymbol{\Omega}_w = \mathbf{H}_w \boldsymbol{\Omega} \mathbf{H}'_w,$$

[1] This point, and many other technical details, are described in the extended version of the paper (see also Di Fonzo and Marini, 2003).

[2] For a discussion, including alternative objective functions on which the benchmarking can be founded, and suggestions to save computational times due the the dimensions of matrices generally involved in this kind of problems, see Di Fonzo and Marini (2003).

$$
\mathbf{y}_w = \begin{pmatrix} \mathbf{z} \\ \mathbf{y}_{0,1} \\ \vdots \\ \mathbf{y}_{0,M-1} \end{pmatrix}, \qquad \mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}.
$$

Guerrero and Nieto (1999) proposed an original procedure according to which the final estimates are obtained through a VAR-based benchmarking. A VAR(q) model $\mathbf{\Pi p} = \mathbf{a}$, where $\mathbf{\Pi}$ is the $(Mn \times Mn)$ matrix containing the autoregressive coefficients and $E(\mathbf{aa}') = \mathbf{I}_{Mn} \otimes \mathbf{\Sigma}$, is estimated[3] for the M preliminary series. Benchmarked estimates are then obtained through expression (1) using

$$
\mathbf{\Omega} = \hat{\mathbf{\Pi}}^{-1}(\mathbf{Q} \otimes \hat{\mathbf{\Sigma}})(\hat{\mathbf{\Pi}}^{-1})',
$$

where $\mathbf{Q}$ is an $(n \times n)$ positive definite matrix to be derived from the data according to a generalized least squares procedure. A compatibility test is finally used to validate the assumption that preliminary and disaggregated estimates share the same VAR model.

4. Applications

The first application is performed using simulated data. According to Guerrero and Nieto (1999), we simulate 44 quarterly observations from the restricted VAR(2) model

$$
\begin{aligned}
y_{1,t} &= 0.02 + 0.5y_{1,t-1} + 0.1y_{2,t-1} + a_{y,1t} \\
y_{2,t} &= 0.03 + 0.4y_{1,t-1} + 0.5y_{2,t-1} + 0.25y_{1,t-2} + a_{y,2t}
\end{aligned},
$$

using a (2×2) error–covariance matrix $\mathbf{\Sigma}$, with $\sigma_{11} = 0.04$, $\sigma_{22} = 0.01$ and $\sigma_{12} = 0$. The temporal and contemporaneous aggregations of $(y_{1,t}, y_{2,t})$ are assumed as our constraints. The same VAR representation, with a larger variance of the disturbances (0.05 and 0.02, respectively), is used to generate the variables $(p_{1,t}, p_{2,t})$, which can be considered as preliminary estimates of $(y_{1,t}, y_{2,t})$. The exercise is replicated with a non-diagonal covariance matrix $\mathbf{\Sigma}^*$, using $\sigma_{12} = 0.005$ instead of zero. In table 1 summary statistics on discrepancies and relative discrepancies between $y_{1,t}$ and $\hat{y}_{1t}$ are presented. The data–based benchmarking always shows better results, particularly when there is contemporaneous correlation between the series.

Table 1: *Performance indicators on simulated data. Series $y_{1,t}$ and $\hat{y}_{1,t}$.*

cov=Σ	relative discrepancies				discrepancies	
	median	min	max	range	mean	std
Mov. Pres. Principle	0.0017	-0.0402	0.0506	0.0908	0.0133	0.0115
Data based	-0.0007	-0.0291	0.0349	0.0640	0.0121	0.0079

cov=Σ^*	relative discrepancies				discrepancies	
	median	min	max	range	mean	std
Mov. Pres. Principle	-0.0001	-0.0952	0.0898	0.1850	0.0328	0.0285
Data based	-0.0011	-0.0327	0.0354	0.0681	0.0168	0.0097

[3]The order q of the VAR is chosen following a likelihood ratio testing scheme.

In the second application, firstly we individually estimate the Italian monthly value added for industry, both total and according to a six-sector disaggregation[4], then we benchmark the sectoral series assuming the monthly total series as contemporaneous constraint to be fulfilled.

Summary statistics in table 2 refer to corrections to preliminary monthly rates of changes induced by the benchmarked estimates. We find a confirmation of the good performances of the data–based procedure in terms of dimension of corrections. Moreover, it should be noted the direct correlation between the size of corrections and the size of the component series (as measured by the average weight in the system, reported on the last column of the table) in the case of Denton–type benchmarking, while for the data–based procedure this is not the case.

Table 2: *Performance indicators on real data. Corrections to monthly rates of changes.*

	Mov. Pres. Principle					Data based					
sect.	med	min	max	range	std	med	min	max	range	std	weight
1	-0.07	-2.13	2.98	5.11	0.80	-0.04	-2.30	2.67	4.98	0.76	0.19
2	-0.08	-1.87	3.05	4.92	0.72	-0.05	-2.38	3.48	5.86	0.87	0.17
3	-0.05	-1.54	2.02	3.56	0.52	-0.06	-1.88	3.76	5.64	0.86	0.12
4	-0.09	-2.55	3.69	6.24	0.92	-0.07	-2.17	3.06	5.23	0.79	0.22
5	-0.07	-1.89	2.57	4.46	0.70	-0.01	-1.13	1.68	2.81	0.48	0.16
6	-0.06	-1.69	2.43	4.12	0.60	-0.05	-2.27	2.92	5.19	0.80	0.14

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[4]We use the standard Chow–Lin approach using the relevant industrial production index as related series. The series, constant prices of year 2000 and seasonally adjusted, span over 1990.01–2002.12.